**2.5 Dielectric slab waveguide** Consider a dielectric slab waveguide which has a thin GaAs layer of thickness 0.2 μm between two AlGaAs layers. The refractive index of GaAs is 3.66 and that of the AlGaAs layers is 3.40. What is the cut-off wavelength beyond which only a single mode can propagate in the waveguide assuming that the refractive index does not vary greatly with the wavelength? If a radiation of wavelength 870 nm (corresponding to bandgap radiation) is propagating in the GaAs layer, what is the penetration of the evanescent wave into the AlGaAs layers? What is the mode field distance of this radiation?

**Solution**

Given $n_1 = 3.66$ (AlGaAs), $n_2 = 3.4$ (AlGaAs), $2a = 2\times10^{-7}$ m or $a = 0.1$ μm, for only a single mode we need

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} < \frac{\pi}{2}$$

$$\therefore \qquad \lambda > \frac{2\pi a\left(n_1^2 - n_2^2\right)^{1/2}}{\frac{\pi}{2}} = \frac{2\pi(0.1\ \mu\text{m})\left(3.66^2 - 3.40^2\right)^{1/2}}{\frac{\pi}{2}} = 0.542\ \mu\text{m}.$$

The cut-off wavelength is 542 nm.

When $\lambda = 870$ nm,

$$V = \frac{2\pi(1\ \mu\text{m})\left(3.66^2 - 3.40^2\right)^{1/2}}{(0.870\ \mu\text{m})} = 0.979 < \pi/2$$

Therefore, $\lambda = 870$ nm is a single mode operation.

For a rectangular waveguide, the fundamental mode has a *mode field distance*

$$2w_o = \text{MFD} \approx 2a\frac{V+1}{V} = (0.2\ \mu\text{m})\frac{0.979+1}{0.979} = 0.404\ \mu\text{m}.$$

The decay constant $\alpha$ of the evanescent wave is given by,

$$\alpha = \frac{V}{a} = \frac{0.979}{0.1\ \mu\text{m}} = 9.79\ (\mu\text{m})^{-1} \text{ or } 9.79\times10^6\ \text{m}^{-1}.$$

The penetration depth

$$\delta = 1/\alpha = 1/\left[9.79\ (\mu\text{m})^{-1}\right] = 0.102\ \mu\text{m}.$$

The penetration depth is half the core thickness.

**2.8 A multimode fiber** Consider a multimode fiber with a core diameter of 100 μm, core refractive index of 1.475 and a cladding refractive index of 1.455 both at 850 nm. Consider operating this fiber at $\lambda = 850$ nm.

**a** Calculate the $V$-number for the fiber and estimate the number of modes.

**b** Calculate the wavelength beyond which the fiber becomes single mode.

**c** Calculate the numerical aperture.

**d** Calculate the maximum acceptance angle.

**e** Calculate the modal dispersion $\Delta\tau$ and hence the bit rate $\times$ distance product given that rms dispersion $\sigma \approx 0.29\Delta\tau$ where $\Delta\tau$ is the full spread.

**Solution**

Given $n_1 = 1.475$, $n_2 = 1.455$, $2a = 100\times10^{-6}$ m or $a = 50$ μm and $\lambda = 0.850$ μm. The $V$-number is,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(50\ \mu\text{m})\left(1.475^2 - 1.455^2\right)^{1/2}}{(0.850\ \mu\text{m})} = \mathbf{89.47}$$

Number of modes $M$,

$$M = \frac{V^2}{2} = \frac{89.47^2}{2} \approx \mathbf{4002}$$

The fiber becomes monomode when,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} < 2.405$$

or

$$\lambda > \frac{2\pi a\left(n_1^2 - n_2^2\right)^{1/2}}{2.405} = \frac{2\pi(50\ \mu\text{m})\left(1.475^2 - 1.455^2\right)^{1/2}}{2.405} = \mathbf{31.6\ \mu m}$$

For wavelengths longer than 31.6 μm, the fiber is a single mode waveguide.

The numerical aperture NA is

$$NA = \left(n_1^2 - n_2^2\right)^{1/2} = \left(1.475^2 - 1.455^2\right)^{1/2} = 0.242$$

If $\alpha_{\max}$ is the maximum acceptance angle, then,

$$\alpha_{\max} = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin(0.242/1) = \mathbf{14°}$$

Modal dispersion is given by

$$\frac{\Delta\tau_{\text{intermode}}}{L} = \frac{n_1 - n_2}{c} = \frac{1.475 - 1.455}{3\times10^8\ \text{m s}^{-1}}$$

$$= 66.7\ \text{ps m}^{-1} \text{ or } \mathbf{67.6\ ns\ per\ km}$$

Given that $\sigma \approx 0.29\Delta\tau$, maximum bit-rate is

$$BL = \frac{0.25L}{\sigma_{\text{total}}} \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{(0.29)(66.7\ \text{ns km}^{-1})}$$

*i.e.* $BL =$ **13 Mb s$^{-1}$ km** (only an estimate!)

We neglected material dispersion at this wavelength which would further decrease $BL$. Material dispersion and modal dispersion must be combined by

$$\sigma_{\text{total}}^2 = \sigma_{\text{intermode}}^2 + \sigma_{\text{material}}^2$$

For example, assuming an LED with a spectral rms deviation $\sigma_\lambda$ of about 20 nm, and a $D_m \approx -200$ ps km$^{-1}$ nm$^{-1}$ (at about 850 nm)we would find

$$\sigma_m = -(200\ \text{ps km}^{-1}\ \text{nm}^{-1})(20\ \text{nm})(1\ \text{km}) \approx 4000\ \text{ps km}^{-1} \text{ or } 4\ \text{ns km}^{-1},$$

which is substantially smaller than the intermode dispersion and can be neglected.

---

**2.9 A single mode fiber** Consider a fiber with a $SiO_2$-13.5%$GeO_2$ core of diameter of 8 μm and refractive index of 1.468 and a cladding refractive index of 1.464 both refractive indices at 1300 nm where the fiber is to be operated using a laser source with a half maximum width of 2 nm.

**a** Calculate the $V$-number for the fiber. Is this a single mode fiber?

**b** Calculate the wavelength below which the fiber becomes multimode.

**c** Calculate the numerical aperture.

**d** Calculate the maximum acceptance angle.

**e** Obtain the material dispersion and waveguide dispersion and hence estimate the bit rate × distance product ($B\times L$) of the fiber.

**Solution**

**a** Given $n_1 = 1.475$, $n_2 = 1.455$, $2a = 8\times10^{-6}$ m or $a = 4$ μm and $\lambda = 1.3$ μm. The $V$-number is,

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} = \frac{2\pi(4\ \mu\text{m})\left(1.468^2 - 1.464^2\right)^{1/2}}{(1.3\ \mu\text{m})} = \mathbf{2.094}$$

**b** Since $V < 2.405$, this is a single mode fiber. The fiber becomes multimode when

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} > 2.405$$

or

$$\lambda < \frac{2\pi a\left(n_1^2 - n_2^2\right)^{1/2}}{2.405} = \frac{2\pi(4\ \mu\text{m})\left(1.468^2 - 1.464^2\right)^{1/2}}{2.405} = \mathbf{1.13\ \mu m}$$

For wavelengths shorter than 1.13 μm, the fiber is a multi-mode waveguide.

**c** The numerical aperture NA is

$$NA = \left(n_1^2 - n_2^2\right)^{1/2} = \left(1.468^2 - 1.464^2\right)^{1/2} = \mathbf{0.108}$$

**d** If $\alpha_{\max}$ is the maximum acceptance angle, then,

$$\alpha_{\max} = \arcsin\left(\frac{NA}{n_o}\right) = \arcsin(0.108/1) = \mathbf{6.2°}$$

so that the *total acceptance angle* is **12.4°.**

**e** At $\lambda = 1.3$ μm, from the figure, $D_m \approx -7.5$ ps km$^{-1}$ nm$^{-1}$, $D_w \approx -5$ ps km$^{-1}$ nm$^{-1}$.

$$\frac{\Delta\tau_{1/2}}{L} = \left|D_m + D_w\right|\Delta\lambda_{1/2}$$

$$= |-7.5-5 \text{ ps km}^{-1}\text{ nm}^{-1}|(2\text{ nm}) = 15 \text{ ps km}^{-1} + 10 \text{ ps km}^{-1}$$

$$= 0.025 \text{ ns km}^{-1}$$

Obviously materials dispersion is **15 ps km$^{-1}$** and waveguide dispersion is **10 ps km$^{-1}$**

The maximum bit-rate distance product is then

$$BL \approx \frac{0.59L}{\Delta\tau_{1/2}} = \frac{0.59}{0.025 \text{ ns km}^{-1}}$$ **= 23.6 Gb s$^{-1}$ km.**

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## 2.14 A graded index fiber

**a** Consider an optimal graded index fiber with a core diameter of 30 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that the fiber is coupled to a laser diode emitter at 1300 nm and a spectral linewidth (FWHM) of 3 nm. Suppose that the material dispersion coefficient at this wavelength is about −5 ps km$^{-1}$ nm$^{-1}$. Calculate, the total dispersion and estimate the bit rate × distance product of the fiber. How does this compare with the performance of a multimode fiber with the same core radius, and $n_1$ and $n_2$? What would be the total dispersion and maximum bit rate if an LED source of spectral width (FWHM) $\Delta\lambda_{1/2} \approx 80$ nm is used?

**b** If $\sigma_{\text{intermode}}(\gamma)$ is the rms dispersion in a graded index fiber with a profile index $\gamma$, and if $\gamma_o$ is the optimal profile index, then

$$\frac{\sigma_{\text{intermode}}(\gamma)}{\sigma_{\text{intermode}}(\gamma_o)} = \frac{2(\gamma - \gamma_o)}{\Delta(\gamma + 2)}$$

and $\gamma_o$ is given by Eq. (2) in § 2.7. Calculate the new dispersion and bit rate × distance product if $\gamma$ is 10% greater than the optimal value $\gamma_o$.

### Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$

Modal dispersion for 1 km of graded index fiber is

$$\sigma_{\text{intermode}} \approx \frac{Ln_1}{20\sqrt{3}c}\Delta^2 = \frac{(1000)(1.474)}{20\sqrt{3}(3\times10^8)}(0.01425)^2$$

$$= 2.9\times10^{-11} \text{ s or } \mathbf{0.029\ ns}$$

The material dispersion (FWHM) is

$$\Delta\tau_{m(1/2)} = LD_m\Delta\lambda_{1/2} = (1000\ \text{m})(-5\ \text{ps ns}^{-1}\ \text{km}^{-1})(3\ \text{nm})$$

$$= \mathbf{0.015\ ns}$$

Assuming a Gaussian output light pulse shape, rms material dispersion is,

$$\sigma_m = 0.425\Delta\tau_{1/2} = (0.425)(0.015 \text{ ns}) = 0.00638 \text{ ns}$$

Total dispersion is

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_m} = \sqrt{0.029^2 + 0.00638^2} = 0.0295\text{ns}.$$

so that $B = 0.25/\sigma_{\text{total}} =$ **8.5 Gb**

If this were a multimode step-index fiber with the same $n_1$ and $n_2$, then the rms dispersion would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{(n_1 - n_2)}{c} = \frac{1.474 - 1.453}{(3\times10^8 \text{ m s}^{-1})}$$

$$= 70 \text{ ps m}^{-1} \text{ or } \mathbf{70\ ns\ per\ km}$$

Maximum bit-rate is

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} \approx \frac{0.25L}{(0.28)\Delta\tau} = \frac{0.25}{(0.28)(70\ \text{ ns km}^{-1})}$$

*i.e.* $BL$ = **12.8 Mb s$^{-1}$ km** (only an estimate!)

The corresponding $B$ for 1 km would be around 13 Mb s$^{-1}$

With LED excitation, again assuming a Gaussian output light pulse shape, rms material dispersion is

$$\sigma_m = (0.425)\Delta\tau_{m(1/2)} = (0.425)LD_m\Delta\lambda_{1/2}$$

$$= (0.425)(1000\ \text{ m})(-5\ \text{ ps ns}^{-1}\ \text{ km}^{-1})(80\ \text{ nm})$$

$$= 0.17 \text{ ns}$$

Total dispersion is

$$\sigma_{\text{total}} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_m} = \sqrt{0.029^2 + 0.17^2} = \mathbf{0.172\ ns}$$

so that $B = 0.25/\sigma_{\text{total}}$ = **1.45 Gb**

The effect of material dispersion now dominates intermode dispersion.

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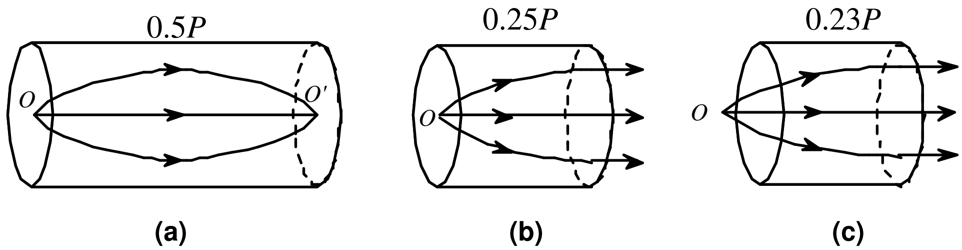
**2.16 GRIN rod lenses** Graded index (GRIN) rod lens is a glass rod whose refractive index changes parabolically from its central axis where the index is maximum. It is like a very thick, short graded index fiber whose diameter is perhaps 0.5 -5 mm. Such GRIN rod lenses of different lengths can be used to focus or collimate light rays as illustrated in Figure 2Q16-1. The principle of operation can be understood by considering ray trajectories in a stratified medium in which ray trajectories are sinusoidal paths. One pitch ($P$) is a full one period variation in the ray trajectory along the rod axis. Figure 2Q16-1 (a), (b) and (c) show half-pitch (0.5$P$), quarter-pitch and (0.25$P$) and 0.23P GRIN rod lenses. The point $O$ in (a) and (b) is on the rod face center where as in (c) it is slightly away from the rod face.

**a** How would you represent Figure 2Q16-1 (a) using two conventional converging lenses. What are $O$ and $O'$?

**b** How would you represent Figure 2Q16-1 (b) using a conventional converging lens. What is $O'$?

**c** Sketch ray paths for a GRIN rod with a pitch between 0.25$P$ and 0.5$P$ starting from $O$ at the face center. Where is $O'$?

**d** What use is 0.23$P$ GRIN rod lens in Figure 2Q16-1 (c)?

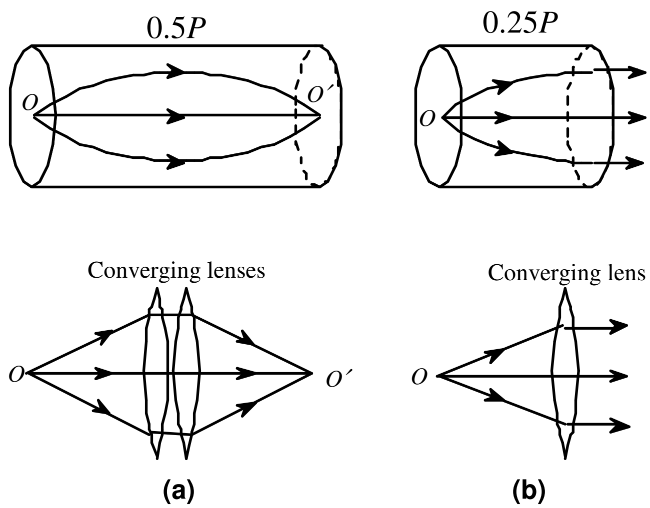



Graded index (GRIN) rod lenses of different pitches. (a) Point $O$ is on the rod face center and the lens focuses the rays onto $O'$ on to the center of the opposite face. (b) The rays from $O$ on the rod face center are collimated out. (c) $O$ is slightly away from the rod face and the rays are collimated out.

**Figure 2Q16-1**

**Solution**

**a,b**

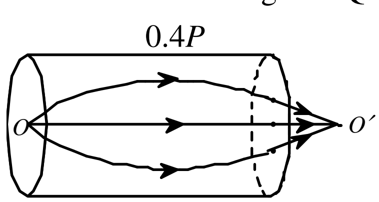



(a) The beam bending from $O$ to $O'$ using a GRIN rod can be achieved equivalently by using two converging lenses. $O$ and $O'$ are the focal points of the lenses (approximately). (Schematic only).

(b) The collimation of rays from a point source on the face of a GRIN rod can be equivalently achieved by a single converging lens whose focal length is 0.25 $P$ and $O$ is the focal point. (Schematic only).

**Figure 2Q16-2**

**c** The rays paths are shown in Figure 2Q16-3. If $O$ is on the rod face, $O'$ is outside the rod.




Ray paths in a GRIN rod that has a pitch between 0.25$P$ to 0.5$P$. (Schematic only.)

**Figure 2Q16-3**

**d** Since the point $O$ does not have to be right on the face of the GRIN rod, it can be used to collimate a point source $O$ by bringing the rod sufficiently close to $O$; a fixed annular spacer can "fix" the required proximity of the rod to $O$. Since the source does not have to be in contact with the face of the rod, possible damage (such as scratches) to the face are avoided.